How Jeremy Bentham would defend against coordinated attacks

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Outline

What do we look at?

- 1 central player (“warden”)
- threat of coordinated attack by N “prisoners”

- warden
  - how much *costly ressources* (“guard level”) to fight off possible attack?
  - what *information* about guard level to release in order to exploit prisoner’s coordination problem? (prison design)

What about Bentham?

- Bentham's suggestion: Panopticon
  - no information on guard level
  - keep prisoners separate (to hamper coordination)
- Bentham's claims
  - coordination to breakout will never be achieved
  - regardless of how many/whether guard(s) are on duty "[...] *so far from it, that a greater multitude than ever were yet lodged in one house might be inspected by a single person*"
  - can be applied to everything: schools, factories, hospitals...

Is this (related to) economics?

- Foucault: enforcement by panopticon allowed "accumulation of men" necessary for industrial take off
- add endogenous information structure to global games (Carlsson and van Damme 1993, Morris and Shin...)
  - central bank defending currency peg against speculators (Morris and Shin 1998)
  - government defending against coup d'état (Chassang and i Miquel 2009)

Main result

- Bentham was right if the number of prisoners is high
    - secrecy of guard level optimally exploits coordination problem
    - in equilibrium warden uses minimal guard level
    - probability of breakout is almost zero nevertheless

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- rough intuition
  - “matching pennies” incentives
  - law of large number: quite precise idea of how many prisoners revolt
    - suppose many
    - employ more guards
    - no one wants to revolt. . . contradiction

Model

- one warden
  - sets a guard level $\gamma \in \Re_+$
  - payoff:
    - $-B - \gamma$ if there is a break out
    - $-\gamma$ if there is no break out
- N prisoners
  - actions: "revolt" (r), "not revolt" (n)
  - payoff:

	break out	no break out
r	$b > 0$	$-q < 0$
n	0	0

- breakout iff strictly more than γ prisoners revolt
- Assumption: $B \geq N + 1$
  (prevent breakout under complete info)

Information

		Guard level observable	
		Yes	No
Coordination problem	No	(1a) Benchmark	(1b) Benchmark
	Yes	(2) Transparency	(3) Panopticon

Table: The four information structures we consider.

Transparency (guard level observed, no coordination)

- if $\gamma \geq N$: not revolt (dominant)
- if $\gamma < 1$: revolt (dominant)
- if $1 \leq \gamma < N$
  - either all revolt in subgame equilibrium
  - or none revolts in subgame equilibrium

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- equilibrium selection as in global games
- result (roughly):
  - play r if and only if $\gamma < \lceil bN/(q+b) \rceil$
  - warden sets $\gamma = \lceil bN/(q+b) \rceil$

Panopticon (guard level unobserved, no coordination) I

- only mixed strategy equilibria
- only prisoner symmetric equilibria
  probability p to revolt
  - number revolting prisoners: binomial distribution

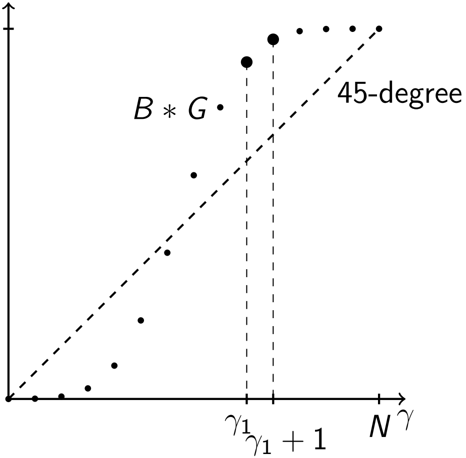
Lemma

In equilibrium, the warden mixes between two adjacent guard levels γ_1 and γ_1+1 where $\gamma_1 \in \{0, \ldots, N-1\}$.

- possibly multiple equilibria

Panopticon (guard level unobserved, no coordination) II

- warden payoff: $-(1-G(\gamma))B-\gamma$ (binomial distrib. is G)



Main Result

Theorem (Bentham was right)

Let N be sufficiently large. Then, the warden mixes between 0 and 1 in the unique equilibrium of the panopticon model. The warden's payoff is higher in this equilibrium than in the transparency model.
In the panopticon, the probability of a breakout is arbitrarily close to zero for sufficiently high N.

Main Result (rough intuition)

- for high N distribution of revolting prisoners G concentrated around mode pN
- around mode marginal utility of $\gamma \uparrow$ high
- γ_1 substantially above mode
- probability that more than γ_1 prisoners revolt low
- prisoner strictly prefers not to revolt

- what is different for $\gamma_1=0$?

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- what is different for $\gamma_1 = 0$?
    - revolt is dominant strategy if $\gamma_1 = 0$
    - 0-1 equilibrium: less coordination game but one-to-one "matching pennies"

Discussion

- How to save a currency peg?
  - keep your foreign currency reserves secret!
  - what about "forward guidance" and transparency?
- Minimal enforcement
  - Bentham and Foucault
  - What about massive police presence at demonstrations/football etc.?

Robustness/Extensions

- payoff when unsuccessfully revolting might depend on guard level
  - revolutions: punishment if seen
  - say $-q - \rho\gamma/N$
  - everything goes through: behave as watched because you might be watched
- payoff of not revolting depends on whether there is a breakout
  - revolution: punishment of non revolting (everything goes through)
  - free riding: can destroy strategic complementarity (destroys results)
- some randomness in breakout probability
  - prob of breakout is $\beta \mathbb{1}_{m>\gamma} + (1-\beta)m/N$
- attackers have different sizes

Conclusion

- coordinated attack model where central player chooses
  - defense level
  - information about defense level
- how to exercise power through the choice of information structure
- optimal to keep defense level secret (for N large etc.)

Benchmark (no coordination problem)

- guard level observed
  - all revolt if $\gamma < N$
  - none revolts otherwise
  - equilibrium: $\gamma = N$
- guard level unobserved
  - either all or none revolt
  - γ either 0 or N
  - mixed strategy equilibrium
- equilibrium payoffs
  - warden: $-N$
  - prisoner: 0

Transparency model (guard level observed, no coordination), details I

- warden chooses guard level with trembling hand $\gamma \sim N(\tilde{\gamma}, \varepsilon')$
- prisoner observes signal drawn from uniform distribution on $[\gamma - \varepsilon, \gamma + \varepsilon]$

Lemma

Let $\varepsilon' > 0$. Assume that $bN/(q+b) \notin \mathbb{N}$ and define

$$\theta^* = \left\lceil \frac{bN}{q+b} \right\rceil .$$

Then for any $\delta > 0$, there exists an $\bar{\varepsilon} > 0$ such that for all $\varepsilon \leq \bar{\varepsilon}$, a player receiving a signal below $\theta^* - \delta$ will play r and a player receiving a signal above $\theta^* + \delta$ will play n.

Transparency model (guard level observed, no coordination) , details II

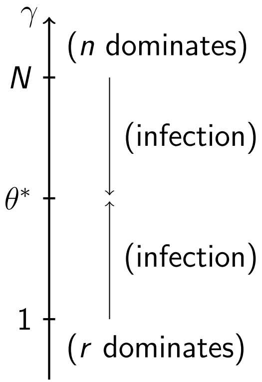

Figure: Infection of beliefs among prisoners

Other results I

Theorem (high disutility of breakout B)

Unless a single guard deters prisoners in the transparency model, the warden is better off in the panopticon if B is sufficiently large.

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Theorem (high disutility of breakout B)

Unless a single guard deters prisoners in the transparency model, the warden is better off in the panopticon if B is sufficiently large.

- only 0-1 equilibrium exists for high B
- any other γ_1:
  - for B high enough, γ_1 is only optimal if p is very low
  - prisoners strictly prefer not to revolt

Other results II

Theorem (incentives to revolt b/q)

For b/q sufficiently high, the warden payoff is $-N$ in all models.

- *Suppose $B^{\frac{N-1}{N}} > N$: Then, for $b/q \in (N-1, B^{\frac{N-1}{N}} - 1)$, the warden's payoff in every equilibrium of the panopticon model is higher than in the equilibrium of the transparency model.*
- *Suppose $N > B^{\frac{N-1}{N}}$: Then, for $b/q \in (B^{\frac{N-1}{N}} - 1, N-1)$, there exists an equilibrium in the panopticon model in which the warden's equilibrium payoff is lower than in the transparency model.*